Wigner crystal and other insulating phases of two-dimensional electrons in high magnetic fields

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A concise review of the history and of the basic physics of the Wigner crystal (electron solid) in two-dimensional electron systems in semiconductor heterostructures is given. The results of our experimental study on the formation and of the properties of the magnetic field-induced insulating phase, supposed to be a Wigner crystal, in a two-dimensional electron gas in In$_{0.55}$Ga$_{0.47}$As/InP heterostructures are surveyed. These structures are characterised by a much greater disorder potential than AlGaAs/GaAs heterostructures due to the inherent alloy disorder in the InGaAs layer supporting the two-dimensional electron gas. In high magnetic fields, below a Landau level filling factor of 0.4-0.5 divergent resistivity, non-linear current-voltage characteristics with a threshold, and a transition from non-activated to activated transport were observed. A model is proposed for the Wigner crystal-like ordering of the two-dimensional electron gas in this system with large disorder.

Keywords: Wigner crystal, In$_{0.55}$Ga$_{0.47}$As/InP, two-dimensional electron gas.

1. Introduction

The occurrence and properties of insulating phases in two-dimensional electron gas (2DEG) in high magnetic field have recently been intensively studied in connection with the still elusive Wigner crystallisation, and also in connection with the possibility of other types of exotic phases, e.g., the Hall insulator. In 1934, the Hungarian born physicist Jenő (Eugene) Wigner [1] noted (in the context of the theory of cohesion of simple free-electron like metals such as sodium) that the electron gas in a conductor would crystallise, forming an electron solid, when the Coulomb energy dominates the kinetic energy of the electrons [2,3] (for early reviews of the theory see e.g. Refs. 4 and 5). In bulk solids the kinetic energy (Fermi energy) and the Coulomb energy of electrons vary as $n^{2/3}$ and $n^{1/3}$, respectively (n is the volume density of electrons), so the crystallisation condition surmised by Wigner would be reached in a sufficiently diluted electron gas. The appearance of this electron solid, named after Wigner, eluded experimental observation for a long time because an appropriate electron system has not been found. The fact that the analogous phase transition in the classical 2DEG might be observed with electrons trapped on the surface of liquid helium was first pointed out by Crandall and Williams [6]. Subsequently Chaplik [7] suggested that a similar crystallisation can occur in the inversion layer near the surface of a semiconductor. Only in the eighties and nineties of the just passing century, – with the emergence of the physics of two-dimensional electron systems, – could the Wigner solidification be observed for the first time in the electrons on the surface of liquid helium [8,9]. Then Wigner crystallisation was found in semiconductor heterostructures, particularly in the GaAlAs/GaAs system [10–16], but not without with an added twist, i.e., by applying a high magnetic field perpendicularly to the plane of the 2DEG to suppress the kinetic energy of electrons, in line with the conjecture of Lozovik and Yudson [17] concerning the possibility of overcoming the experimental barrier represented by the too large kinetic-to-interaction energy ratio. As a matter of fact, the experimental work in the course of which the fractional quantum Hall effect was discovered was originally aimed at searching for signs of the existence of this Wigner crystal in very high magnetic fields in high quality GaAlAs/GaAs heterostructures [18,19].

While in the past decade the bulk of experimental work on the Wigner crystal was carried out on the GaAlAs/GaAs heterostructure, in which the disorder potential is much smaller than the Coulomb potential (small disorder limit), the In$_{0.55}$Ga$_{0.47}$As/InP system (InGaAs/InP for short), where these two potentials can be of the same magnitude, might present new opportunities for the study of this phenomenon. InGaAs/InP and related heterostructures have recently received much attention for their use in optoelectronic and high-speed electronic devices, so studies of basic physical phenomena in these material systems have an important bearing on practical applications, too.

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This paper gives a brief survey of the basic properties and experiments on the Wigner solid in semiconductors (for more details see the reviews in Refs. 14, 16, 20–22) and reviews our recent results on the Wigner solidification in the two-dimensional electron gas in InGaAs/InP heterostructures [23–27]. Our experiments were performed on InGaAs/InP heterostructures with low 2DEG density \((n_s < 2 \times 10^{11} \text{ cm}^{-2})\) and low mobility \(\mu < 6 \times 10^4 \text{ cm}^2/\text{Vs}\). These structures are characterised by a much greater disorder potential than the more conventional AlGaAs/GaAs system, due to the inherent alloy disorder in the InGaAs layer.

2. Wigner crystal in two-dimensional electron gas

The physical mechanism responsible for the formation of the electron solid is connected with the relationship between the kinetic energy of the electrons and the potential energy due to the Coulomb interaction. If the potential energy is sufficiently great, the electrons will be ordered into a configuration having a minimal energy, maximising the mutual distance between them, and form a regular lattice which is a triangular one in 2D [6]. The electron solid will be pinned by the impurities or by the disorder, so the Wigner crystal forms an insulating phase. The electron solid can be “melted” by increasing the temperature. Also in sufficiently high electric field the insulating solid phase can be depinned (analogously to the case of a charge density wave), and electrical conduction emerges.

For a classical two-dimensional electron gas the Wigner crystal occurs when the ratio of the potential and kinetic energies \(\Gamma = <V>/<K>\) is sufficiently great (here \(<V> = (\pi n_s)^{1/2}e^2/(4\pi e_0c)\) and \(<K> = k_B T\). Liquid-solid transition (melting) occurs along the classical phase boundary at \(\Gamma = \Gamma_m\). For the critical value theoretical estimations yield \(\Gamma_m = 100\) [15,20,28–30], while experiments on electrons on the surface of liquid helium [8,9,31] and also on the surface of solid neon [32] gave \(\Gamma_m = 140 \pm 10\). The melting of the 2D electron solid occurs according to the Kosterlitz-Thouless mechanism, i.e., by the dissociation of bound dislocation pairs that are already formed before reaching the melting temperature [9,33,34]. A distinguishing feature of the solid phase is that it supports shear wave propagation, as has been demonstrated experimentally on the 2D electron solid on the surface of liquid helium by Deville et al. [35].

For a degenerate electron gas like those found at semiconductor heterojunctions or in quantum wells the role of the kinetic energy is played by the Fermi energy \(E_F\). Now the formation of the Wigner crystal is favoured at low densities, because of the competition between \(E_F\) which increases proportionally to \(n_s\) in 2D, and the Coulomb potential energy \(<V>\) which increases only as \(n_s^{1/2}\). For high densities, the large value of the Fermi wave vector means that the electrons are delocalised, thus inhibiting the solid formation. The (quantum) order parameter is defined (at \(T = 0\)) as \(r_s = <V>/\pi E_F = e^2m^*/(4\pi e_0e^2R_{\text{BAM}}^2)\sqrt{(\pi n_s)^{1/2}}\) [15,20,22]. For large \(r_s\) the electrons order in a solid phase. As \(r_s\) decreases, at \(r_s = r_s^{\text{crit}}\) the long-range order is lost and the electron solid melts (quantum melting). Melting can be achieved by increasing \(n_s\), i.e., by compression. This is the so-called, Wigner transition. Based on Monte Carlo calculations the theory yields \(r_s^{\text{crit}} = 37\) [36] (c.f. also the reviews in Refs. 15 and 20).

For typical semiconductor 2DEG structures the achievable lowest density is still too high for a Wigner solid to form, however not long ago an insulating phase was detected in GaAlAs/GaAs with exceptionally low two-dimensional hole gas, \(p_s = 5 \times 10^9 \text{ cm}^{-2}\), and the value of the experimentally found \(r_s = 35\) led to the supposition that a Wigner crystallisation was observed [37]. Similar results have also been reported by Simmons et al. [38].

On the other hand, as was pointed out at first by Lozovik and Yudson [17] (see also Refs. 15 and 20) in a sufficiently large magnetic field the kinetic energy of the electrons is quenched and the electrons become localised within a distance of the magnetic length \(l_B = [e/(\hbar c B)]^{1/2}\) which might lead to the formation of the magnetic field induced Wigner solid (MWS). The process is governed by the ratio of the cyclotron energy to the kinetic (Fermi) energy of the electrons, the order parameter being \(\gamma = h_{\text{BAM}}/E_F = eB/\pi n_s = v^2\). Once again for large values of the order parameter \(\gamma\) the electron system is solid, and for small values of \(\gamma\) the system is liquid. At a critical value \(v_c = v_c^{-1}\) the 2DEG undergoes a transition from a quantum liquid to the magnetically induced Wigner solid.

At low temperatures in high magnetic fields, i.e., at small Landau level filling factor \((\nu = n_s/\hbar e/B)\) the ground state of a 2DEG with small disorder is expected to be an electron solid (Wigner crystal) [20]. The formation of the Wigner crystal in high magnetic fields (MWS) has been observed in 2D electrons [10–16] and also in 2D holes [39–41] in nearly perfect GaAlAs/GaAs heterostructures, and recently in the InGaAs/InP heterostructure too [23–27]. For the critical filling factor \(v_c\) theory yields values in the range for 1/3 to 1/11 [15–20]. According to the experiments in GaAlAs/GaAs the critical filling factor \(v_c\) at \(T = 0\) K, below which the Wigner crystallisation occurs, was about 0.2 [10,11,15] and 0.28 [12,13,16]. In our experiments on 2DEG in InGaAs/InP in the case of strong disorder to be discussed below a higher value of \(v_c = 0.4–0.5\) was obtained [25–27].

Experiments on 2DEG in the GaAlAs/GaAs system where the ratio of the disorder potential to the Coulomb energy is \(e(V)/U \ll 1\), indicated that the electron solid exhibits activated [42,43] and non-linear transport properties [10,11,15,20] which can be attributed to the pinning on the random potential introduced by defects and impurities. From the analysis of transport and photoluminescence measurements performed in high magnetic fields the melting
curve and the phase diagram of the MIWS have been deduced [10, 11, 13–16, 20, 44]. Photoluminescence experiments also confirmed that the electron solid has a triangular lattice [45]. Similar transport properties were also observed by the present author and his coworkers [23–27] in the InGaAs/InP system with large disorder potential, where e(V)/U = 1, when the ground state of the 2DEG might be expected to be glass-like, i.e., a Wigner glass [46].

3. Experiments on the Wigner solidification in 2DEG in InGaAs/InP

We performed an experimental study of InGaAs/InP heterostructures with low 2DEG density. These structures are characterised by a much greater disorder potential than the AlGaAs/GaAs heterostructures due to the inherent alloy disorder in the InGaAs layer.

The samples were modulation-doped lattice matched In_{0.53}Ga_{0.47}As/InP heterostructures grown by liquid phase epitaxy [24, 25]. The structure consisted of three layers grown on (100) semi-insulating InP substrates: a 1 μm low-doped (p-type) InP buffer (−10^{15}\, \text{cm}^{-3}), a 0.2–0.5 μm n-doped InP supply layer (2 to 6)×10^{16}\, \text{cm}^{-3}), and a 0.5–1.5 μm low-doped (p-type) In_{0.53}Ga_{0.47}As layer (−10^{15}\, \text{cm}^{-3}). The low-doped layers were grown from melts containing the rare element Sm [47, 48]. The 2DEG was formed in the quasi-triangular potential well in the InGaAs layer at the InGaAs/InP heterointerface. The electron density and mobility were (0.3–2)×10^{11}\, \text{cm}^{-2} and (1–6)×10^{4}\, \text{cm}^{2}/\text{Vs}, respectively. Low temperature mobility of 2DEG in such structures is inherently limited by alloy scattering [49], however, in our samples the contribution from the interface roughness scattering at the heterointerface is also substantial [50].

Samples in the form of double cross Hall bars were prepared by conventional photolithography. Magnetotransport measurements were carried out from 4.2 K down to 40 mK in a He^3–He^4 dilution refrigerator fitted into the bore of a resistive magnet capable of reaching 22 Tesla. Magnetic fields and temperature were estimated to be correct to about 0.02 Tesla and better than a few percent respectively. Persistent photoconductivity was used to control the 2DEG density. Both conventional dc and low-frequency (8–13 Hz) lock-in measuring techniques were used, and both two- and four-terminal resistivity measurements were performed.

Representative low temperature (60 mK) magnetotransport data are shown in Fig. 1. In this sample and also in the other ones the longitudinal resistivity \( \rho_{xx} \) began to increase dramatically with increasing magnetic field below about Landau level filling factor about \( v = 0.5 \). Reaching very large values in comparison with the quantum unit of resistance \( h/e^2 = 26\, \text{kΩ} \). The Hall resistivity \( \rho_{xy} \), after having reached the value corresponding to the \( i = 1 \) quantum Hall plateau, remained constant till the strongly increasing values of \( \rho_{xx} \) prevented the accurate measurement of \( \rho_{xy} \). The appearance of this high resistivity region was accompanied by a noisy resistance roll-off in the low frequency ac measurements and with the onset of a large out-of-phase component of the resistivity. All these signal the formation of an insulating phase (IP) in the 2DEG.

In the insulating phase, below filling factors \( v < 0.5 \) the longitudinal resistivity \( \rho_{xx} \) displayed an activated behaviour for temperatures above about 200 mK (Fig. 2) with a magnetic field dependent activation energy \( E_A \) [26, 27], similarly as observed previously in the GaAlAs/GaAs system [42, 43].

![Fig. 1. Longitudinal (\( \rho_{xx} \)) and Hall (\( \rho_{xy} \)) resistivity versus the magnetic field for a sample with \( n_s = 1.2×10^{11}\, \text{cm}^{-2} \), highlighting the appearance of the insulating phase below about \( v = 0.5 \).](image_url)

![Fig. 2. Longitudinal resistance \( \rho_{xx} \) versus the reciprocal temperature for a sample with \( n_s = 7.5×10^{10}\, \text{cm}^{-2} \). The magnetic fields correspond to range of filling factors from 0.25 to 0.37.](image_url)
Fig. 3. Dependence of the activation energy $E_A$ on the filling factor $v$. 2DEG mobility and density for samples 1 through 3 are $1.6 \times 10^4$, $2.9 \times 10^4$, and $5.2 \times 10^4$ cm$^2$/Vs and $3 \times 10^{10}$, $1.2 \times 10^{11}$, and $7.5 \times 10^{10}$ cm$^{-2}$, respectively. Linear fits indicated correspond to the linear relationship $E_A = E_{A0} - \alpha v$.

The activation energies $E_A$ were determined according to $\rho_{XX} = \rho_0 \exp(E_A/2kT)$. Figure 3 shows that the values of $E_A$ decrease roughly linearly with $v$, with approximately equal slopes for all the samples. The intercepts with the horizontal axis [v$_0$(IP)] mark the onset of the region of activated transport. The higher is the disorder (as indicated by the value of the 2DEG mobility), the larger is the extrapolated activation energy $E_{A0}$ at $v = 0$, and the critical filling factor $v_c$(IP) for the onset of the activated transport.

The data for the onset for various samples at various magnetic fields were extrapolated to infinite magnetic field values (assumed in all theoretical models) using $v = n_e eB$ as suggested in Ref. 42. As shown in Fig. 4 a limiting value of $v_c(B \rightarrow \infty) = 0.37$ was obtained for infinite B, which is about twice the corresponding value ($v_c = 0.19$) obtained for high mobility (low-disorder) GaAlAs/GaAs [42]. This extrapolated value of $v_c$ is identified as the critical filling factor for the appearance of the Wigner solid in the 2DEG [42].

In the IP, two-terminal I–V characteristics (Figs. 5 and 6) revealed a non-linearity with a filling factor and temperature dependent “plateau” and “threshold” as well as a characteristic shift in the phase between the current and voltage in the measurements of the longitudinal resistivity $\rho_{XX}$ [23–25,27]. The observed threshold field increased with the magnetic field, i.e., with decreasing filling factor (Fig. 5), and with decreasing temperature (Fig. 6). For higher filling factors these features were absent. These observations, like in the case of the GaAlAs/GaAs system in the literature [10,11,15,20], can be considered as an indication of the formation of a magnetic-field-induced insulating phase below filling factors of about 0.5. The insulating phase is thought to be a MIWS pinned by the strong disorder. Its conduction mechanism is governed by the pinning-depinning phenomena.

Varying the magnetic field, the temperature and the concentration of electrons via persistent photoconductivity, a part of the phase diagram corresponding to this transition was determined. Extrapolating the low temperature parts of the threshold voltage versus temperature curves to $v = 0$, the critical temperatures for the disappearance of the non-linearity and of the supposed pinning-depinning transi-

Fig. 4. Extrapolation of the onset $v(E_A)$ to infinite B using $v = n_e eB$.

Fig. 5. Two-terminal I–V characteristics at 70 mK in a sample with $n_e = 1.2 \times 10^{11}$ cm$^{-2}$. The filling factors are $v = 0.414, 0.382, 0.354, 0.331, 0.310$, and $0.292$, respectively.
4. Discussion of the experiments on the InGaAs/InP system

All these features have previously been observed in low disorder AlGaAs/GaAs heterostructures, and treated as an evidence for the formation of the MIWS. Therefore it is concluded that our results demonstrate the possibility of the formation of magnetic field induced Wigner solid (Wigner glass?) in 2DEG in the case of strong disorder too. The insulating phase, i.e., the Wigner solid, is pinned by the strong disorder and its conduction mechanism is governed by the pinning-depinning transition.

The threshold electric fields (a few tens of V/cm) are much larger than expected for an ordinary Wigner crystal. This may be explained by assuming the pinning of the crystalline electron solid on the large disorder. The strength of the disorder, characterised with the broadening of the Landau levels deduced from the decay of the amplitude of the Shubnikov-de Haas oscillations in low magnetic fields, is about $eV > 8-10$ meV, comparable in magnitude to the Coulomb energy $U$, $eV/U = 1$. This disorder is much stronger than found in GaAlAs/GaAs, where this ratio can be smaller than 1/100. The incompressible fluid state of the 2DEG is fully deformed and a glass-like condensate (Wigner glass, Ref. 46) pinned by the disorder is formed. The stronger the disorder the more glassy the Wigner solid pinned by the disorder. In low mobility samples a strongly pinned Wigner solid is expected with the onset of activated transport at higher values of $V$ than in the high mobility case. Indeed, as shown in Fig. 8, the extrapolated activation energies $E_{\text{As}}$ in the IP obtained here for 2DEG in the high-disorder InGaAs/InP system and in the literature for 2DEG in the low disorder GaAlAs/GaAs system [42,43] show a surprisingly good correlation with the 2DEG mobility, with a logarithmic slope of -0.5. This means a direct correlation with the disorder which is quantified by the Landau level width obtained in the self consistent Born approximation for short range scatterers $\Gamma = (\hbar/\mu)^{1/2}$ [30].

In our previous paper [27] a model has been proposed for the Wigner crystal-like ordering of the electrons in the presence of strong disorder. In the InGaAs/InP material system studied here the strong disorder is due to the inherent alloy disorder and to the roughness of the heterointerface [49,50]. Both disorder potentials are characterised by a short range. The range of the alloy disorder potential is comparable to the lattice constant. Previous investigations have also shown [51] that in our material system the characteristic period of the heterointerface roughness is $L = 5$ to 7 nm. Both characteristic distances are less than the lattice constant of the Wigner crystal, i.e., $L < a_W = (\pi a_0)^{-5} = 15-30$ nm in our case. So, the following scenario is possible: long range disorder ($L > a_W$) prevents the Wigner crystallisation and leads to the usual single particle localisation, while in the case of short range disorder ($L < a_W$),
We observed diverging resistance, transition from non-activated to activated transport, and non-linear I-V characteristics with filling factor-dependent thresholds in 2DEG in InGaAs/InP in high magnetic fields at fractional filling factors. The observations were interpreted as the formation of a glass-like Wigner solid, and a model has been proposed to the ordering of electrons in the presence of a strong short-range disorder.

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**References**


**5. Conclusions**

To sum up, a short review of the history and basic physics of the Wigner crystal was given. Experimental results concerning the formation of the Wigner solid in the 2DEG in InGaAs/InP heterostructures were reviewed and the properties of the supposed Wigner crystal were discussed.


