Diffraction pattern conversion as a method to exhibit the average droplet diameter of fuel aerosol

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1. Introduction

Measurement of the fuel aerosol droplet diameters is very difficult because of the fast change of their sizes caused by their evaporation and association. On the other hand, the profound knowledge of aerosol streams is necessary for designers of new fuel burners and for diagnostics. In such cases the diffraction method is one of the most useful measuring procedures. The droplet size of monodispersion medium is easy to recognise but analysis of diffraction pattern of polydispersion medium is more difficult. This work presents a method which enables to measure the average aerosol diameter $\bar{D}_p$ in real-time process.

2. Principles of diffraction methods used to measure droplet diameter

The small aerosol droplets cause the Fraunhofer diffraction of the plane monochromatic light wave (Fig.1). If the droplet sizes are identical it will cause an Airy pattern. Light intensity I(r,D) in this pattern is given by [1]

$$I(r,D) = I_0 \left| \frac{2J_1\left(\frac{\pi Dr}{\lambda f}\right)}{\frac{\pi Df}{\lambda f}} \right|^2 \tag{1}$$

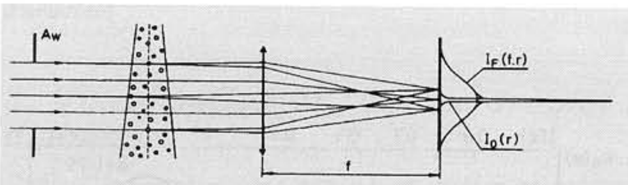

Fig.1. Fraunhofer diffraction pattern of plane monochromatic wave light caused by an aerosol

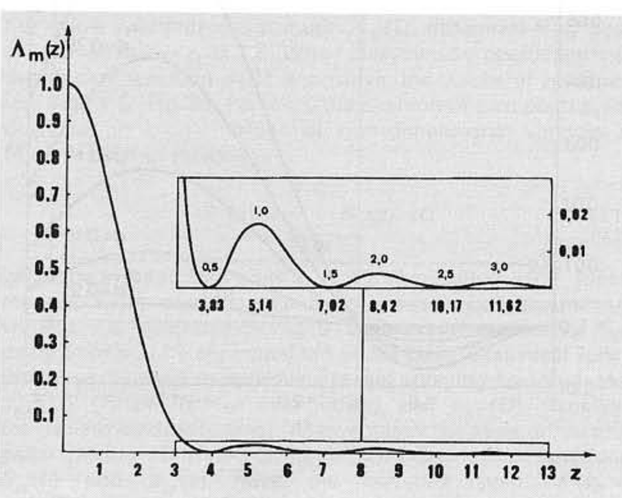

Fig. 2. Non-dimensional light intensity $\Lambda_w(z)$ of monodispersion medium

where: r – radius vector, λ – wavelength, f – focal length, J_1 Bessel function. Introducing the non-dimensional variable

$$z = \frac{\pi Dr}{\lambda f} \tag{2}$$

and dividing (Eq.1) by I_0 gives the non-dimensional light intensity $\Lambda_w(z)$ (Fig.2)

$$\Lambda_w(z) = \left| \frac{2J_1(z)}{z} \right|^2 \tag{3}$$

Next dark and bright rings appear for variable z

$$\begin{aligned} z_{0,5} &= 3.832,\ z_{1.0} = 5.136,\ z_{1.5} = 7.016 \\ z_{2.0} &= 8.417,\ z_{2.5} = 10.17,\ z_{3.0} = 11.62 \end{aligned} \tag{4}$$

Finding positions of next relative extremes r_i from diffraction pattern you obtain well known relations [2]

$$D \cong 1.22\frac{\lambda f}{r_{0.5}} \cong 1.64\frac{\lambda f}{r_{1.0}} \cong 2.23\frac{\lambda f}{r_{1.}} \cong \ldots \tag{5}$$

If the droplet sizes are not identical, the light intensity of the polydispersion medium is given by

$$I(r) = \int_0^\infty \rho(D)\Lambda\left(\frac{\pi Dr}{\lambda f}\right) dD \tag{6}$$

where $\rho(D)$ is a statistical distribution of droplet diameters. As a measure of difference between monodispersion and polydispersion medium the parameter d is accepted [3]

$$d = \frac{\sigma}{\bar{D}_\rho} \tag{7}$$

where σ is a standard deviation and $\bar{D}_\rho$ is the average value of distribution $\rho(D)$. Fig. 3 shows three various Gaussian distributions having the same value of parameter d and the same non dimensional light intensity $I_G(z)$ shown in Fig. 5. As it is

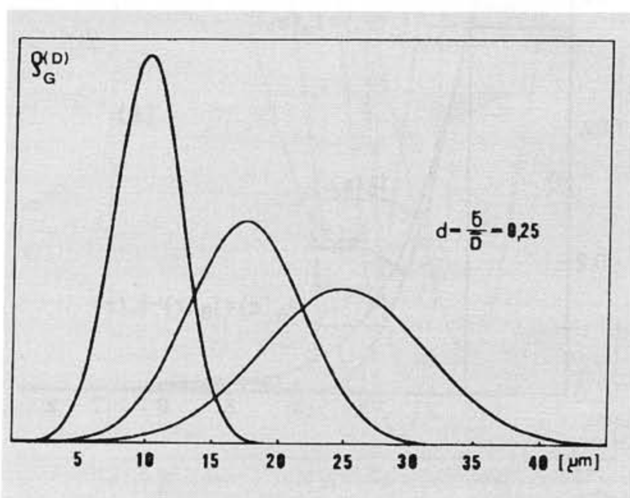

Fig. 3. Three Gaussian distributions of droplet diameters with the same value of relative standard deviation $d = 0.25$

shown in Fig. 4 the raise of relative standard deviation d changes the diffraction patterns of aerosol streams. The Airy rings gradually disappear and when $d \geqslant 0.155$ there are no black and bright rings in sight. In this situation Eq. 5 becomes completely useless. Fig. 5 shows the light intensity $I_G(z)$ of polydispersion medium described by Gaussian distribution with $d=0.25$ and the light intensity $I_m(z)$ of monodispersion medium which droplet diameter is D_m. Of course

$$\bar{D}_G = D_m \text{ and } z = \frac{\pi \bar{D}_G r}{\lambda f} = \frac{\pi D_m r}{\lambda f} \tag{8}$$

For non-dimensional variable $z = 0 \div 1.6$ light intensity $I_G(z)$ is considerably bigger than $I_m(z)$.

Fig. 6 illustrates the difference $R(z)$ between light intensity $I_G(z)$ and $I_m(z)$

$$R(z) = I_G(z) - I_m(z) \tag{9}$$

This difference attains extreme value for $z \cong 1$ and $z \cong 3$. For $z \cong 1.5$ the difference $R(z) \cong 0$ does not depend on value of d. It has been remarked that the type of distribution $\rho(D)$ has only a slight influence on light intensity $I(z)$. The results of detailed analysis confirm this observation but only for symmetrical distributions. That means

$$\rho(D) = \rho(2\bar{D} - D) \tag{10}$$

For unsymmerical distributions $\rho(D)$ function $R(z)$ depends considerably on the type of this distribution (Fig.7,8). The functions $R(z) = I_G(z) - I_m(z)$ are almost identical in the range of non dimensional variable $z = 0 \div 1.5$. Light intensity $I(z)$ in the "upper part" ($I(z) > 0.55$, $z < 1.5$) depends only on value of relative standard deviation d (Fig. 7). In the "lower part" of $I(z)$ ($I(z) < 0.55$, $z > 1.5$) functions $R(z)$ are almost identical for symmetrical distributions $\rho_G(D)$ and $\rho_r(D)$ whereas $R(z)$ for unsymmetrical distribution $\rho_l(D)$ differs considerably (Fig.7). Essential conclusion of this analysis is that the non dimensional light intensity $I(z)$ for $z \to 1.5$ is nearly independent of relative standard deviation d and type of statistical distribution $\rho(D)$ as a result of the difference $R(z) \to 0$. For that reason point z_0 contains "pure" information about average droplet diameter $\bar{D}_\rho = D_m$

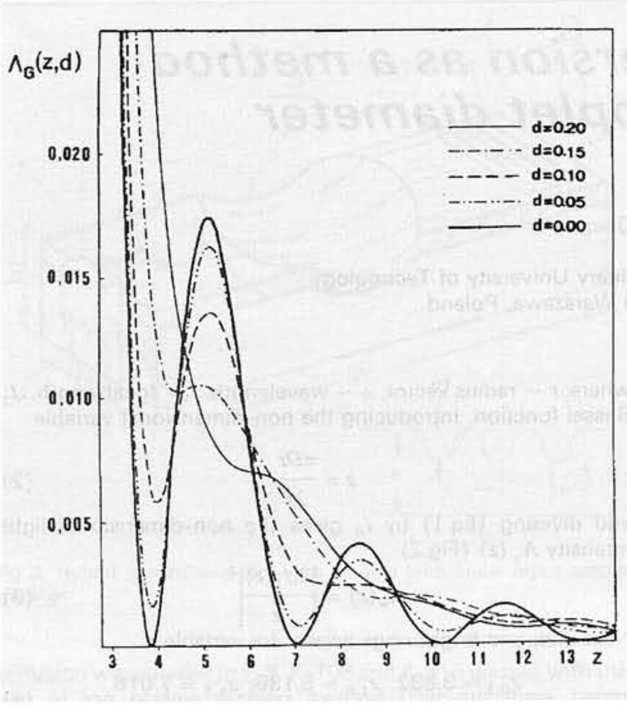

Fig. 4. Airy rings disappearing as a result of relative standard deviation growth for Gaussian distribution $\rho_G(\bar{D}, d)$

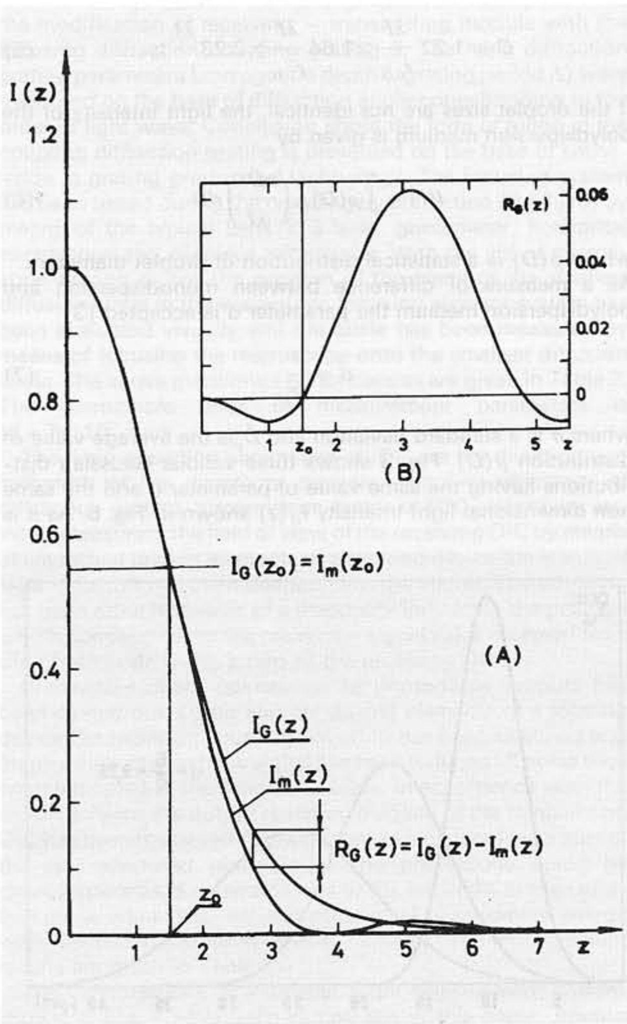

Fig. 5. Light intensity $I_G(z)$ of polydispersion medium described by Gaussian distribution with relative standard deviation $d = 0.25$ together with the light intensity $I_m(z)$ of monodispersion medium

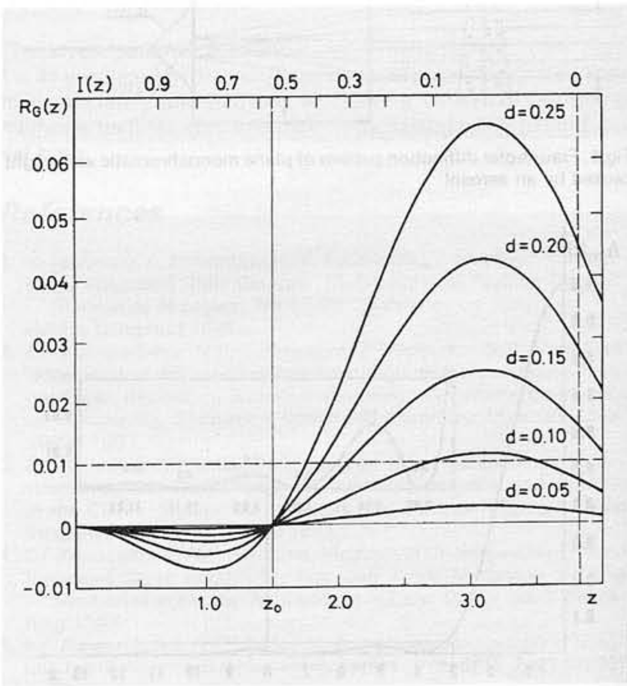

Fig. 6. Difference $R(z)$ between light intensity $I_G(z)$ for Gaussian distribution polydispersion medium and light intensity $I_m(z)$ for monodispersion medium

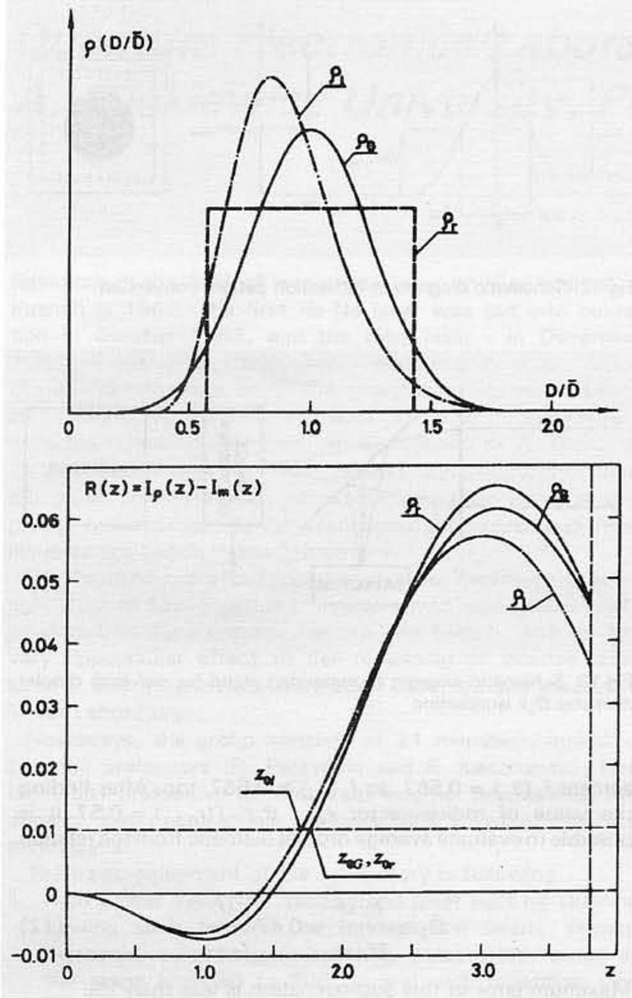

Fig. 7. Difference $R(z)$ for three aerosol droplet diameter distributions with the same average value $\bar{D}_\rho$ and relative standard deviation $d = 0.25$ (ρ_G – Gaussian distribution, ρ_l – log-normal distribution, ρ_r – uniform distribution)

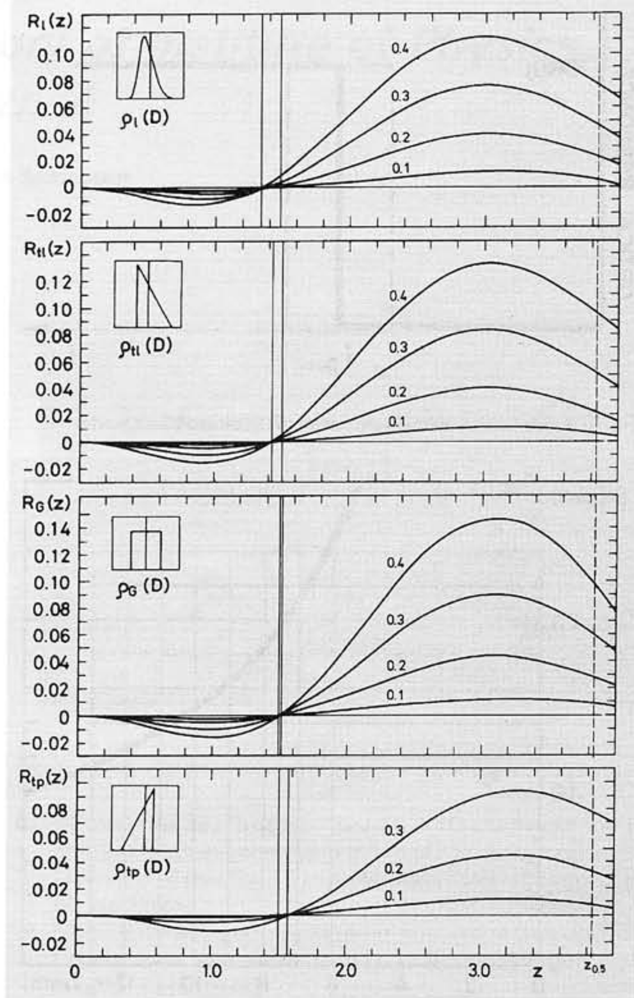

Fig.8. Functions $R_\rho(z) = I_\rho(z) - I_m(z)$ for four statistical distributions $\rho(D)$ ($\rho_l(D)$ – long-norm distribution, $\rho_{tl}(D)$ – triangular distribution with $\gamma > 0$, $\rho_r(D)$ – uniform distribution, $\rho_{tp}(D)$ – triangular distribution with $\gamma < 0$)

3. Visualisation of average droplet diameter

3.1. Characteristic point of diffraction light intensity $I(z)$

For whole symmetrical distributions $\rho(D)$ difference $R(z)$ goes to zero when $z \to z_0 \cong 1.5$. When dissymmetry coefficient γ of distribution function $\rho(D)$ is positive, the values of $z_0(d)$ are less than 1.5 (Fig. 8). For $\gamma < 0$ the position of zero point $z_0(d)$ displaces to bigger values of non-dimensional variable z. Minimisation of integral

$$S_\rho(z) = \frac{1}{d_k}\int_0^{d_k} |R_\rho(z)|\,dD \qquad (11)$$

gives the average zero point $z_{0\,\min}$ for distribution $\rho(D)$. Mean module $S_\rho(z)$ of difference $R_\rho(z)$ versus non dimensional variable z is illustrated in Fig. 9. Detailed information of five distributions $\rho(D)$ are contained in the table. Statistical functions usually used to approximate real spraying spectrum like $\rho_{RR}(D)$ (Rosin-Ramler distribution) and $\rho_{NT}(D)$ (Nukiyama-Tanasawa distribution) [4] give nearly the same diffraction pattern as log-norm distribution $\rho_l(D)$. Extreme error functions $S_{pl}(z)$ and $S_{tp}(z)$ have the common point $K(\bar{z}_0 = 1.47,\ S_{pl}(\bar{z}_0) \approx 0.002)$. Maximum value of difference $R_\rho(\bar{z}_0, d)$ in this point is less then 0.01 for whole type of statistical distributions $\rho(D)$. Light intensity of monodispersion

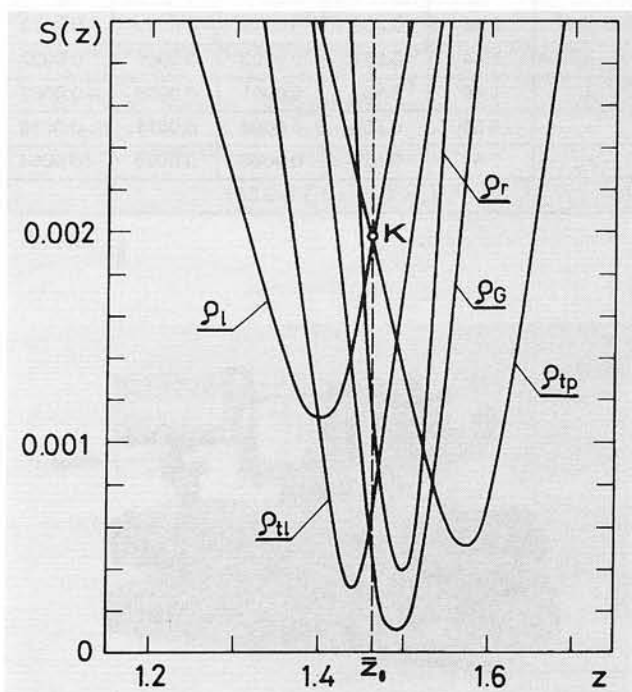

Fig. 9. Mean module $S_\rho(z)$ of difference $R_\rho(z)$ versus nondimensional variable

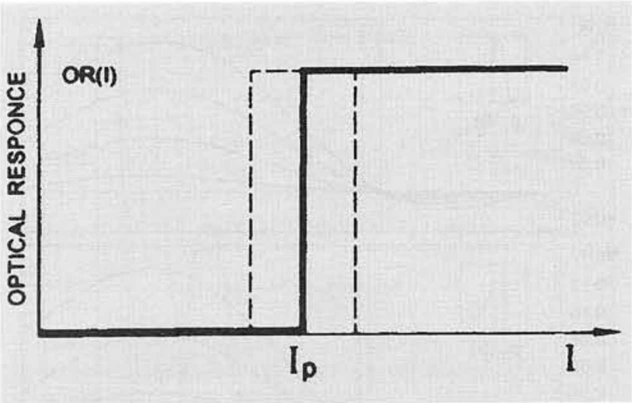

Fig.10. Rectangular optical response $OR(I)$ of image converter

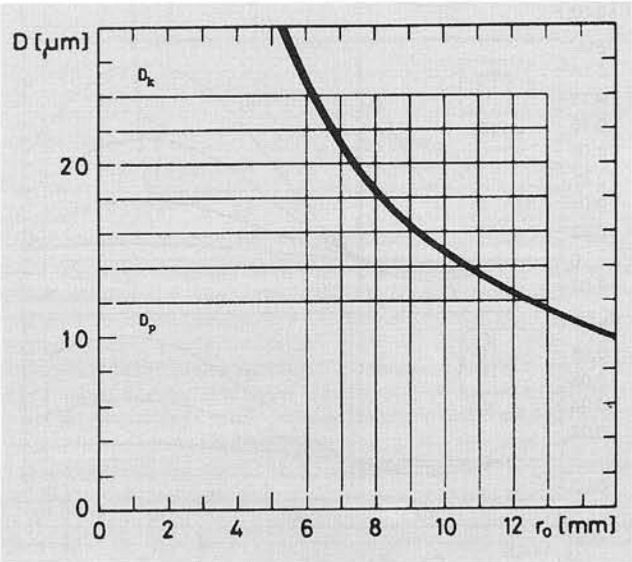

Fig.11. Average droplet diameter of $\bar{D}$ versus radius-vector $r_{0.57}$ of converted diffraction picture (for $\lambda = 0.63$ μm and $f = 0.5$ m)

$\rho(D)$	$z_{0\,\min}$	$I(z_{0\,\min})$	$S(z_{0\,\min})$	$S(\bar{z}_0)$	$R(\bar{z}_0)_{\max}$
ρ_I	1.40	0.599	0.0011	0.0020	0.0073
ρ_{II}	1.44	0.581	0.0003	0.0006	0.0032
ρ_G	1.49	0.558	0.0001	0.0005	−0.0007
ρ_r	1.50	0.553	0.0004	0.0011	−0.0030
ρ_{tp}	1.57	0.521	0.0005	0.0020	−0.0064
$\bar{z}_0 = 1.47$ $I(\bar{z}_0) = 0.567$					

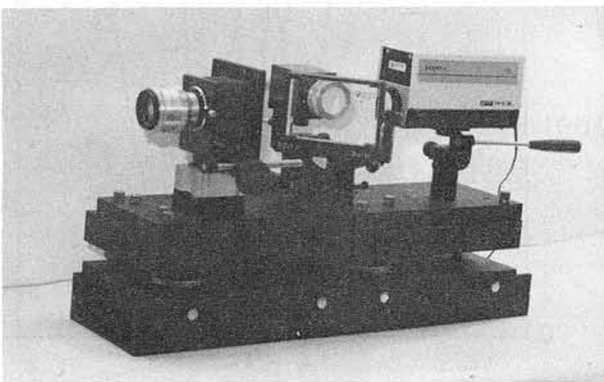

Fig.14. Measuring stand with TV camera

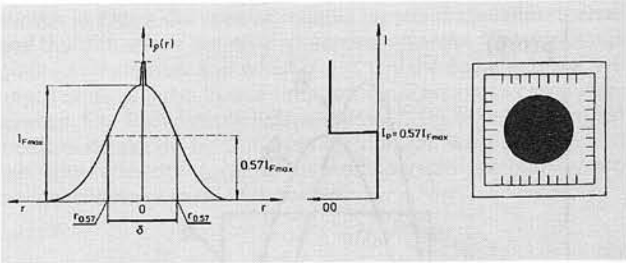

Fig.12. Schematic diagram of diffraction pattern conversion

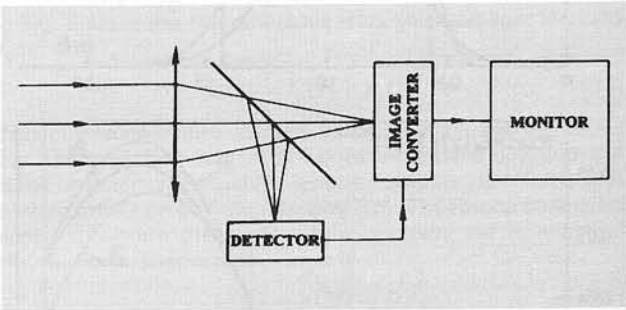

Fig.13. Schematic diagram of measuring stand for real-time droplet diameter $\bar{D}_\rho v$ isualisation

aerosol $I_m(\bar{z}_0) = 0.567$, so $I_\rho(\bar{z}_0) \cong 0.567$, too. After finding the value of radius-vector $r_{0.57}$ that $I(r_{0.57}) = 0.57$ it is possible to evaluate average droplet diameter from the relation

$$\bar{D}_\rho \cong \frac{\bar{z}_0 \lambda f}{\pi r_{0.57}} = 0.47 \frac{\lambda f}{r_{0.57}} \tag{12}$$

Maximum error of this approximation is less than 1%.

3.2. Conclusions

After using image converter with a rectangular optical response $OR(I)$, the Airy picture $I(r)$ becomes a ring. The radius of that ring depends on threshold light intensity I_p of the characteristic $OR(I)$ (Fig.9). If $I_p = 0.57 I_{max}$, the radius of converted Airy picture will be $r = r_{0.57}$ (Fig.11). After introducing non-linear scale $\bar{D} = f(r_{0.57}$ (Fig.10) image converter becomes an analogue measuring instrument (Fig.12). This idea has been successfully tested with TV and CCD cameras (Fig.13). Whole formulas to evaluate average droplet diameter $\bar{D}_\rho$ are derived under many conditions [1÷3, 5÷8]. The most important assumption is that light beam deflects only once on the droplet of aerosol stream.

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